

## NAG C Library Function Document

### nag\_prob\_von\_mises (g01erc)

#### 1 Purpose

nag\_prob\_von\_mises (g01erc) returns the probability associated with the lower tail of the von Mises distribution between  $-\pi$  and  $\pi$ .

#### 2 Specification

double nag\_prob\_von\_mises (double t, double vk, NagError \*fail)

#### 3 Description

The von Mises distribution is a symmetric distribution used in the analysis of circular data. The lower tail area of this distribution on the circle with mean direction  $\mu_0 = 0$  and concentration parameter kappa,  $\kappa$ , can be written as

$$\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2\pi I_0(\kappa)} \int_{-\pi}^{\theta} e^{\kappa \cos \Theta} d\Theta,$$

where  $\theta$  is reduced modulo  $2\pi$  so that  $-\pi \leq \theta < \pi$  and  $\kappa \geq 0$ . Note that if  $\theta = \pi$  then nag\_prob\_von\_mises (g01erc) returns a probability of 1. For very small  $\kappa$  the distribution is almost the uniform distribution, whereas for  $\kappa \rightarrow \infty$  all the probability is concentrated at one point.

The method of calculation for small  $\kappa$  involves backwards recursion through a series expansion in terms of modified Bessel functions, while for large  $\kappa$  an asymptotic Normal approximation is used.

In the case of small  $\kappa$  the series expansion of  $\Pr(\Theta \leq \theta : \kappa)$  can be expressed as

$$\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2} + \frac{\theta}{2\pi} + \frac{1}{\pi I_0(\kappa)} \sum_{n=1}^{\infty} n^{-1} I_n(\kappa) \sin n\theta,$$

where  $I_n(\kappa)$  is the modified Bessel function. This series expansion can be represented as a nested expression of terms involving the modified Bessel function ratio  $R_n$ ,

$$R_n(\kappa) = \frac{I_n(\kappa)}{I_{n-1}(\kappa)}, \quad n = 1, 2, 3, \dots,$$

which is calculated using backwards recursion.

For large values of  $\kappa$  (see Section 7) an asymptotic Normal approximation is used. The angle  $\Theta$  is transformed to the nearly Normally distributed variate  $Z$ ,

$$Z = b(\kappa) \sin \frac{\Theta}{2},$$

where

$$b(\kappa) = \frac{\sqrt{\frac{2}{\pi}} e^{\kappa}}{I_0(\kappa)}$$

and  $b(\kappa)$  is computed from a continued fraction approximation. An approximation to order  $\kappa^{-4}$  of the asymptotic normalizing series for  $z$  is then used. Finally the Normal probability integral is evaluated.

For a more detailed analysis of the methods used see Hill (1977).

#### 4 References

Mardia K V (1972) *Statistics of Directional Data* Academic Press

Hill G W (1977) Algorithm 518: Incomplete Bessel function  $I_0$ : The Von Mises distribution *ACM Trans. Math. Software* **3** 279–284

## 5 Parameters

- 1: **t** – double *Input*  
*On entry:* the observed von Mises statistic,  $\theta$ , measured in radians.
- 2: **vk** – double *Input*  
*On entry:* the concentration parameter  $\kappa$ , of the von Mises distribution.  
*Constraint:* **vk**  $\geq 0$ .
- 3: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_REAL

On entry, **vk** =  $\langle value \rangle$ .  
Constraint: **vk**  $\geq 0.0$ .

### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

nag\_prob\_von\_mises (g01erc) uses one of two sets of constants depending on the value of *machine precision*. One set gives an accuracy of six digits and uses the Normal approximation when **vk**  $\geq 6.5$ , the other gives an accuracy of 12 digits and uses the Normal approximation when **vk**  $\geq 50$ .

## 8 Further Comments

Using the series expansion for small  $\kappa$  the time taken by nag\_prob\_von\_mises (g01erc) increases linearly with  $\kappa$ ; for larger  $\kappa$ , for which the asymptotic Normal approximation is used, the time taken is much less.

If angles outside the region  $-\pi \leq \theta < \pi$  are used care has to be taken in evaluating the probability of being in a region  $\theta_1 \leq \theta \leq \theta_2$  if the region contains an odd multiple of  $\pi$ ,  $(2n + 1)\pi$ . The value of  $F(\theta_2; \kappa) - F(\theta_1; \kappa)$  will be negative and the correct probability should then be obtained by adding one to the value.

## 9 Example

Four values from the von Mises distribution along with the values of the parameter  $\kappa$  are input and the probabilities computed and printed.

## 9.1 Program Text

```

/* nag_prob_von_mises (g01erc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double p, t, vk;
    Integer exit_status, i__, n;
    NagError fail;

    INIT_FAIL(fail);
    exit_status = 0;

    Vprintf("%s\n\n", "g01erc Example Program Results");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");
    Vscanf("%ld%*[\n] ", &n);

    for (i__ = 1; i__ <= n; ++i__)
    {
        Vscanf("%lf%lf%*[\n] ", &t, &vk);
        p = g01erc(t, vk, &fail);
        if (fail.code == NE_NOERROR)
        {
            Vprintf(" p = %10.4f\n", p);
        }
        else
        {
            Vprintf("Error from g01erc.\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
    }
    END:
    return exit_status;
}

```

## 9.2 Program Data

```

g01erc Example Program Data
4
7.0 0.0
2.8 2.4
1.0 1.0
-1.4 1.3

```

## 9.3 Program Results

```

g01erc Example Program Results

p =      0.6141
p =      0.9983
p =      0.7944
p =      0.1016

```