NAG C Library Function Document

nag_deviates_studentized_range (g01fmc)

1 Purpose

nag_deviates_studentized_range (g01fmc) returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic.

2 Specification

```
double nag_deviates_studentized_range (double p, double v, Integer ir,
    NagError *fail)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left(r \int_{-\infty}^{\infty} \phi(y)(\Phi(y) - \Phi(y - qx))^{r-1}\, dy \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

For a given probability p_0, the deviate q_0 is found as the solution to the equation

$$P(q_0; v, r) = p_0, \tag{1}$$

using a root finding procedure. Initial estimates are found using the approximation given in Lund and Lund (1983) and a simple search procedure.

4 References

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32 (2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

1: **p** – double *Input*

On entry: the lower tail probability for the Studentized range statistic, p_0.

Constraint: $0.0 < \mathbf{p} < 1.0$.

2: **v** – double *Input*

On entry: the number of degrees of freedom, v.

Constraint: $\mathbf{v} \geq 1.0$.

3: **ir** – Integer *Input*

On entry: the number of groups, r.

Constraint: $\mathbf{ir} \geq 2$.

4: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **ir** = ⟨*value*⟩.
Constraint: **ir** ≥ 2.

NE_ACCURACY

Warning - There is some doubt as to whether full accuracy has been achieved.

NE_INIT_ESTIMATE

Unable to find initial estimate.

NE_REAL

On entry, $\mathbf{p} \leq 0.0$ or $\mathbf{p} \geq 1.0$: **p** = ⟨*value*⟩.

On entry, **v** = ⟨*value*⟩.
Constraint: $\mathbf{v} \geq 1.0$.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The returned solution, q_*, to equation (1) is determined so that at least one of the following criteria apply.

(a) $|P(q_*; v, r) - p_0| \leq 0.000005$

(b) $|q_0 - q_*| \leq 0.000005 \times \max(1.0, |q_*|)$.

8 Further Comments

To obtain the factors for Duncan's multiple-range test, equation (1) has to be solved for p_1, where $p_1 = p_0^{r-1}$, so on input **p** should be set to p_0^{r-1}.

9 Example

Three values of p, ν and r are read in and the Studentized range deviates or quantiles are computed and printed.

9.1 Program Text

```
/* nag_deviates_studentized_range (g01fmc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{

  /* Scalars */
  double p, v, valq;
  Integer exit_status, i__, ir;
  NagError fail;

  exit_status = 0;

  INIT_FAIL(fail);
  Vprintf("g01fmc Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vprintf("\n%s\n\n",   "  p     v    ir    Quantile ");
  for (i__ = 1; i__ <= 3; ++i__)
    {

      Vscanf("%lf%lf%ld%*[^\n] ",   &p,  &v,  &ir);

      valq = g01fmc(p, v, ir, &fail);
      if (fail.code == NE_NOERROR || fail.code == NE_ACCURACY)
        {
            Vprintf("%5.2f%2s%4.1f%1s%3ld%1s%10.4f\n",    p,  "",  v,  "",  ir,
"",  valq);
        }
      else
        {
          Vprintf("Error from g01fmc.\n%s\n",   fail.message);
          exit_status = 1;
          goto END;
        }
    }

 END:
return exit_status;
}
```

9.2 Program Data

```
g01fmc Example Program Data
0.95 10.0  5
0.3  60.0 12
0.9   5.0  4
```

9.3 Program Results

```
g01fmc Example Program Results

  p     v    ir    Quantile

 0.95  10.0   5     4.6543
 0.30  60.0  12     2.8099
 0.90   5.0   4     4.2636
```