NAG C Library Function Document

nag_prob_non_central_chi_sq (g01gcc)

1 Purpose

nag_prob_non_central_chi_sq (g01gcc) returns the probability associated with the lower tail of the non-central χ^2 distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
```

```
double nag_prob_non_central_chi_sq (double x, double df, double lambda,
          double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the non-central χ^2 distribution with ν degrees of freedom and non-centrality parameter λ, $P(X \leq x : \nu; \lambda)$, is defined by

$$P(X \leq x : \nu; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} P(X \leq x : \nu + 2j; 0) \tag{1}$$

where $P(X \leq x : \nu + 2j; 0)$ is a central χ^2 with $\nu + 2j$ degrees of freedom.

The value of j at which the Poisson weight, $e^{-\lambda/2} \frac{(\lambda/2)^j}{j!}$, is greatest is determined and the summation (1) is made forward and backward from that value of j.

The recursive relationship:

$$P(X \leq x : a + 2; 0) = P(X \leq x : a; 0) - \frac{(x^a/2)e^{-x/2}}{\Gamma(a+1)} \tag{2}$$

is used during the summation in (1).

4 Parameters

1: **x** – double *Input*

On entry: the deviate from the non-central χ^2 distribution with ν degrees of freedom and non-centrality parameter λ.

Constraint: **x** ≥ 0.0.

2: **df** – double *Input*

On entry: the degrees of freedom, ν, of the non-central χ^2 distribution.

Constraint: **df** ≥ 0.0.

3: **lambda** – double *Input*

On entry: the non-centrality parameter, λ, of the non-central χ^2 distribution.

Constraint: **lambda** ≥ 0.0 if **df** > 0.0 or **lambda** > 0.0 if **df** $= 0.0$.

4: **tol** – double *Input*

On entry: the required accuracy of the solution. If nag_prob_non_central_chi_sq is entered with **tol** greater than or equal to 1.0 or less than $10\times$***machine precision***, then the value of $10\times$***machine precision*** is used instead.

5: **max_iter** – Integer *Input*

On entry: the maximum number of iterations to be performed.

Suggested value: 100. See Section 6 for further discussion.

Constraint: **max_iter** ≥ 1.

6: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL_ARG_LT

On entry, **df** must not be less than 0.0: **df** = <*value*>.
On entry, **lambda** must not be less than 0.0: **lambda** = <*value*>.
On entry, **x** must not be less than 0.0: **x** = <*value*>.

NE_2_REAL_ARG_CONS

On entry, **df** = <*value*> while **lambda** = <*value*>.
These parameters must satisfy **lambda** > 0.0 if **df** = 0.0.

NE_INT_ARG_LT

On entry, **max_iter** must not be less than 1: **max_iter** = <*value*>.

NE_POISSON_WEIGHT

The initial value of the Poisson weight used in the summation of (1) (see Section 3) was too small to be calculated. The computed probability is likely to be zero.

NE_CONV

The solution has failed to converge in <*value*> iterations, consider increasing **max_iter** or **tol**.

NE_TERM_LARGE

The value of a term required in (2) (see Section 3) is too large to be evaluated accurately. The most likely cause of this error is both **x** and **lambda** are too large.

NE_CHI_PROB

The calculations for the central chi-square probability has failed to converge. A larger value of **tol** should be used.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

The number of terms in (1) (see Section 3) required for a given accuracy will depend on the following factors:

(i) The rate at which the Poisson weights tend to zero. This will be slower for larger values of λ.

(ii) The rate at which the central χ^2 probabilities, tend to zero. This will be slower for larger values of ν and x.

6.1 Accuracy

The summations described in Section 3 are made until an upper bound on the truncation error relative to the current summation value is less than **tol**.

6.2 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

7 See Also

None.

8 Example

Values from various non-central χ^2 distributions are read, the lower-tail probabilities calculated, and all these values printed out, until the end of data is reached.

8.1 Program Text

```
/* nag_prob_non_central_chi_sq (g01gcc) Example Program.
 *
 * Copyright 1999 Numerical Algorithms Group.
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{

  double df, prob, lambda, tol, x;
  Integer max_iter;
  Integer exit_status=0;
  NagError fail;

  INIT_FAIL(fail);
  Vprintf("g01gcc Example Program Results\n\n");

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");

  Vprintf("\n     x       df      lambda   prob\n\n\n");
  tol = 5e-6;
  max_iter = 50;
```

```
  while ((scanf(" %lf %lf %lf %*[^\n] ", &x, &df, &lambda)) != EOF)
    {

      prob = g01gcc(x, df, lambda, tol, max_iter, &fail);
      if (fail.code == NE_NOERROR)
 {
   Vprintf("%8.3f %8.3f %8.3f %8.4f\n", x, df, lambda, prob);
 }
      else
 {
   Vprintf("Error from g01gcc.\n%s\n", fail.message);
          exit_status = 1;
   goto END;
 }
    }
 END:
  return exit_status;
}
```

8.2 Program Data

```
g01gcc Example Program Data
  8.26   20.0   3.5            :x df lambda
  6.2     7.5   2.0            :x df lambda
 55.76   45.0   1.0            :x df lambda
```

8.3 Program Results

```
g01gcc Example Program Results


     x        df       lambda   prob


   8.260   20.000    3.500   0.0032
   6.200    7.500    2.000   0.2699
  55.760   45.000    1.000   0.8443
```