NAG C Library Function Document

nag_poisson_ci (g07abc)

1 Purpose

nag_poisson_ci (g07abc) computes a confidence interval for the mean parameter of the Poisson distribution.

2 Specification

```
void nag_poisson_ci (Integer n, double xmean, double clevel, double *tl, double *tu,
     NagError *fail)
```

3 Description

Given a random sample of size n, denoted by $x_1, x_2, \ldots, x_n$, from a Poisson distribution with probability function

$$p(x) = e^{-\theta} \frac{\theta^x}{x!}, \quad x = 0, 1, 2, \ldots$$

the point estimate, $\hat{\theta}$, for θ is the sample mean, $\bar{x}$.

Given n and $\bar{x}$ this function computes a $100(1 - \alpha)\%$ confidence interval for the parameter θ, denoted by $[\theta_l, \theta_u]$, where α is in the interval (0,1).

The lower and upper confidence limits are estimated by the solutions to the equations

$$e^{-n\theta_l} \sum_{x=T}^{\infty} \frac{(n\theta_l)^x}{x!} = \frac{\alpha}{2},$$

$$e^{-n\theta_u} \sum_{x=0}^{T} \frac{(n\theta_u)^x}{x!} = \frac{\alpha}{2},$$

where $T = \sum_{i=1}^{n} x_i = n\hat{\theta}$.

The relationship between the Poisson distribution and the χ^2 distribution (see page 112 of Hastings and Peacock (1975)) is used to derive the equations

$$\theta_l = \frac{1}{2n} \chi^2_{2T,\alpha/2},$$

$$\theta_u = \frac{1}{2n} \chi^2_{2T+2,1-\alpha/2},$$

where $\chi^2_{\nu,p}$ is the deviate associated with the lower tail probability p of the χ^2 distribution with ν degrees of freedom.

In turn the relationship between the χ^2 distribution and the gamma distribution (see page 70 of Hastings and Peacock (1975)) yields the following equivalent equations;

$$\theta_l = \frac{1}{2n} \gamma_{T,2;\alpha/2},$$

$$\theta_u = \frac{1}{2n} \gamma_{T+1,2;1-\alpha/2},$$

where $\gamma_{\alpha,\beta;\delta}$ is the deviate associated with the lower tail probability, δ, of the gamma distribution with shape parameter α and scale parameter β. These deviates are computed using nag_deviates_gamma_dist (g01ffc).

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworths

Snedecor G W and Cochran W G (1967) *Statistical Methods* Iowa State University Press

5 Parameters

1: **n** – Integer *Input*

On entry: the sample size, n.

Constraint: $\mathbf{n} \geq 1$.

2: **xmean** – double *Input*

On entry: the sample mean, $\bar{x}$.

Constraint: $\mathbf{xmean} \geq 0.0$.

3: **clevel** – double *Input*

On entry: the confidence level, $(1 - \alpha)$, for two-sided interval estimate. For example $\mathbf{clevel} = 0.95$ gives a 95% confidence interval.

Constraint: $0.0 < \mathbf{clevel} < 1.0$.

4: **tl** – double * *Output*

On exit: the lower limit, θ_l, of the confidence interval.

5: **tu** – double * *Output*

On exit: the upper limit, θ_u, of the confidence interval.

6: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} > 0$.

NE_CONVERGENCE

When using the relationship with the gamma distribution the series to calculate the gamma probabilities has failed to converge.

NE_REAL

On entry, $\mathbf{clevel} \leq 0.0$ or $\mathbf{clevel} \geq 1.0$: $\mathbf{clevel} = \langle value \rangle$.

On entry, $\mathbf{xmean} = \langle value \rangle$.
Constraint: $\mathbf{xmean} \geq 0.0$.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For most cases the results should have a relative accuracy of $\max(0.5\text{e}{-12}, 50.0 \times \epsilon)$ where ϵ is the ***machine precision***. Thus on machines with sufficiently high precision the results should be accurate to 12 significant digits. Some accuracy may be lost when $\alpha/2$ or $1 - \alpha/2$ is very close to 0.0, which will occur if **clevel** is very close to 1.0. This should not affect the usual confidence intervals used.

8 Further Comments

None.

9 Example

The following example reads in data showing the number of noxious weed seeds and the frequency with which that number occurred in 98 sub-samples of meadow grass. The data is taken from page 224 of Snedecor and Cochran (1967). The sample mean is computed as the point estimate of the Poisson parameter θ. nag_poisson_ci (g07abc) is then called to compute both a 95% and a 99% confidence interval for the parameter θ.

9.1 Program Text

```
/* nag_poisson_ci (g07abc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg07.h>

int main(void)
{

  /* Scalars */
  double clevel, sum, tl, tu, xmean;
  Integer exit_status, i, ifreq, n, num;
  NagError fail;

  INIT_FAIL(fail);
  exit_status = 0;
  Vprintf("g07abc Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  /* Read in the number of Noxious Seeds in a sub sample and
   * the frequency with which that number occurs.
   */

  /* Compute the sample mean */
  sum = 0.0;
  n = 0;

  while (scanf("%ld%ld%*[^\n] ", &num, &ifreq) != EOF)
    {
      sum += (double) num * (double) ifreq;
      n += ifreq;
    }
  xmean = sum / (double) n;

  Vprintf("\n");
  Vprintf("The point estimate of the Poisson parameter = %6.4f\n", xmean);
  for (i = 1; i <= 2; ++i)
```

```
    {
      if (i == 1)
        {
          clevel = 0.95;
          Vprintf("\n");
          Vprintf("95 percent Confidence Interval for the estimate\n");
        }
      else
        {
          clevel = 0.99;
          Vprintf("99 percent Confidence Interval for the estimate\n");
        }
      g07abc(n, xmean, clevel, &tl, &tu, &fail);
      if (fail.code != NE_NOERROR)
        {
          Vprintf("Error from g07abc.\n%s\n", fail.message);
          exit_status = 1;
          goto END;
            }

      Vprintf("( %6.4f , %6.4f )\n", tl, tu);
      Vprintf("\n");
    }

 END:
  return exit_status;
}
```

9.2 Program Data

```
g07abc Example Program Data
0 3
1 17
2 26
3 16
4 18
5 9
6 3
7 5
8 0
9 1
10 0
```

9.3 Program Results

```
g07abc Example Program Results

The point estimate of the Poisson parameter = 3.0204

95 percent Confidence Interval for the estimate
( 2.6861 , 3.3848 )

99 percent Confidence Interval for the estimate
( 2.5874 , 3.5027 )
```