

# NAG C Library Function Document

## nag\_rank\_regsn (g08rac)

### 1 Purpose

nag\_rank\_regsn (g08rac) calculates the parameter estimates, score statistics and their variance-covariance matrices for the linear model using a likelihood based on the ranks of the observations.

### 2 Specification

```
void nag_rank_regsn (Nag_OrderType order, Integer ns, const Integer nv[],
    const double y[], Integer p, const double x[], Integer pdx, Integer idist,
    Integer nmax, double tol, double parvar[], Integer pdparvar, Integer irank[],
    double zin[], double eta[], double vapvec[], double parest[], NagError *fail)
```

### 3 Description

Analysis of data can be made by replacing observations by their ranks. The analysis produces inference for regression parameters arising from the following model.

For random variables  $Y_1, Y_2, \dots, Y_n$  we assume that, after an arbitrary monotone increasing differentiable transformation,  $h(\cdot)$ , the model

$$h(Y_i) = x_i^T \beta + \epsilon_i \quad (1)$$

holds, where  $x_i$  is a known vector of explanatory variables and  $\beta$  is a vector of  $p$  unknown regression coefficients. The  $\epsilon_i$  are random variables assumed to be independent and identically distributed with a completely known distribution which can be one of the following: Normal, logistic, extreme value or double-exponential. In `<?:make_boo` an estimate for  $\beta$  is proposed as  $\beta = MX^T a$  with estimated variance-covariance matrix  $M$ . The statistics  $a$  and  $M$  depend on the ranks  $r_i$  of the observations  $Y_i$  and the density chosen for  $\epsilon_i$ .

The matrix  $X$  is the  $n$  by  $p$  matrix of explanatory variables. It is assumed that  $X$  is of rank  $p$  and that a column or a linear combination of columns of  $X$  is not equal to the column vector of 1 or a multiple of it. This means that a constant term cannot be included in the model `<?:make_boo`. The statistics  $a$  and  $M$  are found as follows. Let  $\epsilon_i$  have pdf  $f(\epsilon)$  and let  $g = -f'/f$ . Let  $W_1, W_2, \dots, W_n$  be order statistics for a random sample of size  $n$  with the density  $f(\cdot)$ . Define  $Z_i = g(W_i)$ , then  $a_i = E(Z_{r_i})$ . To define  $M$  we need  $M^{-1} = X(B - A)X$ , where  $B$  is an  $n$  by  $n$  diagonal matrix with  $B_{ii} = E(g'(W_{r_i}))$  and  $A$  is a symmetric matrix with  $A_{ij} = \text{cov}(Z_{r_i}, Z_{r_j})$ . In the case of the Normal distribution, the  $Z_1 < \dots < Z_n$  are standard Normal order statistics and  $E(g'(W_i)) = 1$ , for  $i = 1, 2, \dots, n$ .

The analysis can also deal with ties in the data. Two observations are adjudged to be tied if  $Y_i - Y_j < \text{tol}$ , where **tol** is a user-supplied tolerance level.

Various statistics can be found from the analysis:

- The score statistic  $Xa$ . This statistic is used to test the hypothesis  $H_0 : \beta = 0$ , see (e).
- The estimated variance-covariance matrix  $X(B - A)X$  of the score statistic in (a).
- The estimate  $\beta = MX^T a$ .
- The estimated variance-covariance matrix  $M = (X(B - A)X)^{-1}$  of the estimate  $\beta$ .
- The  $\chi^2$  statistic  $Q = \beta M^{-1} \beta = aX(X(B - A)X)^{-1} Xa$  used to test  $H_0 : \beta = 0$ . Under  $H_0$ ,  $Q$  has an approximate  $\chi^2$  distribution with  $p$  degrees of freedom.
- The standard errors  $M_{ii}^{1/2}$  of the estimates given in (c).
- Approximate  $z$ -statistics, i.e.,  $Z_i = \beta_i / \text{se}(\beta_i)$  for testing  $H_0 : \beta_i = 0$ . For  $i = 1, 2, \dots, n$ ,  $Z_i$  has an approximate  $N(0, 1)$  distribution.

In many situations, more than one sample of observations will be available. In this case we assume the model

$$h_k(Y_k) = X_k T \beta + e_k, \quad k = 1, 2, \dots, \mathbf{ns},$$

where  $\mathbf{ns}$  is the number of samples. In an obvious manner,  $Y_k$  and  $X_k$  are the vector of observations and the design matrix for the  $k$ th sample respectively. Note that the arbitrary transformation  $h_k$  can be assumed different for each sample since observations are ranked within the sample.

The earlier analysis can be extended to give a combined estimate of  $\beta$  as  $\beta = Dd$ , where

$$D^{-1} = \sum_{k=1}^{\mathbf{ns}} X_k T (B_k - A_k) X_k$$

and

$$d = \sum_{k=1}^{\mathbf{ns}} X_k T a_k,$$

with  $a_k$ ,  $B_k$  and  $A_k$  defined as  $a$ ,  $B$  and  $A$  above but for the  $k$ th sample.

The remaining statistics are calculated as for the one sample case.

## 4 References

Pettitt A N (1982) Inference for the linear model using a likelihood based on ranks *J. Roy. Statist. Soc. Ser. B* **44** 234–243

## 5 Parameters

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order = Nag\_RowMajor**. See <?:make\_bo for a more detailed explanation of the use of this parameter.  
*Constraint:* **order = Nag\_RowMajor** or **Nag\_ColMajor**.
- 2: **ns** – Integer *Input*  
*On entry:* the number of samples.  
*Constraint:* **ns**  $\geq 1$ .
- 3: **nv[ns]** – Integer *Input/Output*  
*On entry:* the number of observations in the  $i$ th sample, for  $i = 1, 2, \dots, \mathbf{ns}$ .  
*On exit:* used as internal workspace prior to being restored and hence is unchanged.  
*Constraint:* **nv**[ $i$ ]  $\geq 1$  for  $i = 0, 1, \dots, \mathbf{ns} - 1$ .
- 4: **y[dim]** – const double *Input*  
**Note:** the dimension,  $dim$ , of the array **y** must be at least  $\sum_{i=0}^{\mathbf{ns}-1} \mathbf{nv}[i]$ .  
*On entry:* the observations in each sample. Specifically, **y**( $\sum_{k=1}^i k - 1 \mathbf{nv}[k - 1] + j$ ) must contain the  $j$ th observation in the  $i$ th sample.
- 5: **p** – Integer *Input*  
*On entry:* the number of parameters to be fitted.  
*Constraint:* **p**  $\geq 1$ .

- 6: **x**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **x** must be at least  $\max(1, \mathbf{pdx} \times \mathbf{p})$  when **order** = **Nag\_ColMajor** and at least  $\max(1, \mathbf{pdx} \times \sum_{i=0}^{ns-1} \mathbf{nv}[i])$  when **order** = **Nag\_RowMajor**.  
 If **order** = **Nag\_ColMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*j* – 1) × **pdx** + *i* – 1] and if **order** = **Nag\_RowMajor**, the (*i*, *j*)th element of the matrix *X* is stored in **x**[(*i* – 1) × **pdx** + *j* – 1].  
*On entry:* the design matrices for each sample. Specifically, **x**( $\sum k = 1i - 1\mathbf{nv}[k - 1] + j, l$ ) must contain the value of the *l*th explanatory variable for the *j*th observation in the *i*th sample.  
*Constraint:* **x** must not contain a column with all elements equal.
- 7: **pdx** – Integer *Input*  
*On entry:* the stride separating matrix row or column elements (depending on the value of **order**) in the array **x**.  
*Constraints:*  
     if **order** = **Nag\_ColMajor**, **pdx** ≥  $\sum_{i=0}^{ns-1} \mathbf{nv}[i]$ ;  
     if **order** = **Nag\_RowMajor**, **pdx** ≥ **p**.
- 8: **idist** – Integer *Input*  
*On entry:* the error distribution to be used in the analysis as follows:  
**idist** = 1  
     Normal.  
**idist** = 2  
     Logistic.  
**idist** = 3  
     Extreme value.  
**idist** = 4  
     Double-exponential.  
*Constraint:*  $1 \leq \mathbf{idist} \leq 4$ .
- 9: **nmax** – Integer *Input*  
*On entry:* the value of the largest sample size.  
*Constraint:* **nmax** =  $\max_{1 \leq i \leq ns} (\mathbf{nv}[i - 1])$  and **nmax** > **p**.
- 10: **tol** – double *Input*  
*On entry:* the tolerance for judging whether two observations are tied. Thus, observations *Y*<sub>*i*</sub> and *Y*<sub>*j*</sub> are adjudged to be tied if *Y*<sub>*i*</sub> – *Y*<sub>*j*</sub> < **tol**.  
*Constraint:* **tol** > 0.0.
- 11: **parvar**[*dim*] – double *Output*  
**Note:** the dimension, *dim*, of the array **parvar** must be at least  $\max(1, \mathbf{pdparvar} \times \mathbf{p})$  when **order** = **Nag\_ColMajor** and at least  $\max(1, \mathbf{pdparvar} \times \mathbf{p} + 1)$  when **order** = **Nag\_RowMajor**.  
 Where **PARVAR**(*i*, *j*) appears in this document, it refers to the array element **parvar**[(*j* – 1) × **pdparvar** + *i* – 1] when **order** = **Nag\_ColMajor** and at least **parvar**[(*i* – 1) × **pdparvar** + *j* – 1] when **order** = **Nag\_RowMajor**.

*On exit:* the variance-covariance matrices of the score statistics and the parameter estimates, the former being stored in the upper triangle and the latter in the lower triangle. Thus for  $1 \leq i \leq j \leq \mathbf{p}$ ,  $\mathbf{PARVAR}(i, j)$  contains an estimate of the covariance between the  $i$ th and  $j$ th score statistics. For  $1 \leq j \leq i \leq \mathbf{p} - 1$ ,  $\mathbf{PARVAR}(i + 1, j)$  contains an estimate of the covariance between the  $i$ th and  $j$ th parameter estimates.

12: **pdparvar** – Integer *Input*

*On entry:* the stride separating matrix row or column elements (depending on the value of **order**) in the array **parvar**.

*Constraints:*

if **order** = **Nag\_ColMajor**, **pdparvar**  $\geq \mathbf{p} + 1$ ;  
if **order** = **Nag\_RowMajor**, **pdparvar**  $\geq \mathbf{p}$ .

13: **irank[nmax]** – Integer *Output*

*On exit:* for the one sample case, **irank** contains the ranks of the observations.

14: **zin[nmax]** – double *Output*

*On exit:* for the one sample case, **zin** contains the expected values of the function  $g(\cdot)$  of the order statistics.

15: **eta[nmax]** – double *Output*

*On exit:* for the one sample case, **eta** contains the expected values of the function  $g'(\cdot)$  of the order statistics.

16: **vapvec[dim]** – double *Output*

**Note:** the dimension,  $dim$ , of the array **vapvec** must be at least  $\mathbf{nmax} \times (\mathbf{nmax} + 1)/2$ .

*On exit:* for the one sample case, **vapvec** contains the upper triangle of the variance-covariance matrix of the function  $g(\cdot)$  of the order statistics stored column-wise.

17: **parest[dim]** – double *Output*

**Note:** the dimension,  $dim$ , of the array **parest** must be at least  $4 \times \mathbf{p} + 1$ .

*On exit:* the statistics calculated by the routine as follows. The first  $\mathbf{p}$  components of **parest** contain the score statistics. The next  $\mathbf{p}$  elements contain the parameter estimates. **parest**[ $2 \times \mathbf{p}$ ] contains the value of the  $\chi^2$  statistic. The next  $\mathbf{p}$  elements of **parest** contain the standard errors of the parameter estimates. Finally, the remaining  $\mathbf{p}$  elements of **parest** contain the  $z$ -statistics.

18: **fail** – NagError \* *Input/Output*

The NAG error parameter (see the <?:make\_boo).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **ns** =  $\langle value \rangle$ .

Constraint: **ns**  $\geq 1$ .

On entry, **p** =  $\langle value \rangle$ .

Constraint: **p**  $\geq 1$ .

On entry, **pdx** =  $\langle value \rangle$ .

Constraint: **pdx**  $> 0$ .

On entry, **pdparvar** =  $\langle value \rangle$ .

Constraint: **pdparvar**  $> 0$ .

On entry, **idist** is outside the range 1 to 4: **idist** =  $\langle value \rangle$ .

**NE\_INT\_2**

On entry, **pdx** =  $\langle value \rangle$ , **p** =  $\langle value \rangle$ .

Constraint: **pdx**  $\geq$  **p**.

On entry, **pdparvar** =  $\langle value \rangle$ , **p** =  $\langle value \rangle$ .

Constraint: **pdparvar**  $\geq$  **p** + 1.

On entry, **pdparvar** =  $\langle value \rangle$ , **p** =  $\langle value \rangle$ .

Constraint: **pdparvar**  $\geq$  **p**.

On entry, **pdx** < the sum of **nv**[*i*]: **pdx** =  $\langle value \rangle$ , sum **nv**[*i*] =  $\langle value \rangle$ .

On entry, **nmax**  $\leq$  **p**: **nmax** =  $\langle value \rangle$ , **p** =  $\langle value \rangle$ .

**NE\_INT\_ARRAY**

On entry, **nv**[*i*] =  $\langle value \rangle$ .

Constraint: **nv**[*i*]  $\geq$  1 for  $i = 0, \dots, \mathbf{ns} - 1$ .

**NE\_INT\_ARRAY\_ELEM\_CONS**

*M* elements of array **nv** are less than or equal to zero: *M* =  $\langle value \rangle$ .

**NE\_MAT\_ILL\_DEFINED**

The matrix  $X^T(B - A)X$  is either singular or non-positive-definite.

**NE\_OBSERVATIONS**

All the observations were adjudged to be tied.

**NE\_REAL**

On entry, **tol** =  $\langle value \rangle$ .

Constraint: **tol** > 0.0.

**NE\_REAL\_ARRAY\_ELEM\_CONS**

On entry, all elements in column  $\langle value \rangle$  of **x** are equal to  $\langle value \rangle$ .

**NE\_SAMPLE**

The largest sample size is  $\langle value \rangle$  which is not equal to **nmax**, **nmax** =  $\langle value \rangle$ .

**NE\_ALLOC\_FAIL**

Memory allocation failed.

**NE\_BAD\_PARAM**

On entry, parameter  $\langle value \rangle$  had an illegal value.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

**7 Accuracy**

The computations are believed to be stable.

## 8 Further Comments

The time taken by the routine depends on the number of samples, the total number of observations and the number of parameters fitted.

In extreme cases the parameter estimates for certain models can be infinite, although this is unlikely to occur in practice. See <?:make\_boo for further details.

## 9 Example

A program to fit a regression model to a single sample of 20 observations using two explanatory variables. The error distribution will be taken to be logistic.

### 9.1 Program Text

```

/* nag_rank_regsn (g08rac) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg08.h>

int main(void)
{
    /* Scalars */
    double tol;
    Integer exit_status, i, idist, p, j, nmax, ns, nsum;
    Integer pdx, pdparvar;
    NagError fail;
    Nag_OrderType order;

    /* Arrays */
    double *eta=0, *parest=0, *parvar=0, *vapvec=0, *x=0, *y=0, *zin=0;
    Integer *irank=0, *nv=0;

#ifdef NAG_COLUMN_MAJOR
#define X(I,J) x[(J-1)*pdx + I - 1]
#define PARVAR(I,J) parvar[(J-1)*pdparvar + I - 1]
    order = Nag_ColMajor;
#else
#define X(I,J) x[(I-1)*pdx + J - 1]
#define PARVAR(I,J) parvar[(I-1)*pdparvar + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("g08rac Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");

    /* Read number of samples, number of parameters to be fitted,
     * error distribution parameter and tolerance criterion for ties.
     */
    Vscanf("%ld%ld%ld%lf%*[\n] ", &ns, &p, &idist, &tol);

    /* Allocate memory to nv only */
    if ( !(nv = NAG_ALLOC(ns, Integer)) )
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
    }

```

```

    goto END;
}

Vprintf("\n");
Vprintf("Number of samples =%2ld\n", ns);
Vprintf("Number of parameters fitted =%2ld\n", p);
Vprintf("Distribution =%2ld\n", idist);
Vprintf("Tolerance for ties =%8.5f\n", tol);

/* Read the number of observations in each sample. */

for (i = 1; i <= ns; ++i)
    Vscanf("%ld", &nv[i - 1]);
Vscanf("%*[\n] ");

nmax = 0;
nsum = 0;
for (i = 1; i <= ns; ++i)
{
    nsum += nv[i - 1];
    nmax = MAX(nmax, nv[i - 1]);
}
if (nmax > 0 && nmax <= 100 && nsum > 0 && nsum <= 100)
{
    /* Allocate memory */
    if ( !(eta = NAG_ALLOC(nmax, double)) ||
        !(parest = NAG_ALLOC(4*p+1, double)) ||
        !(parvar = NAG_ALLOC((p+1)*p, double)) ||
        !(vapvec = NAG_ALLOC(nmax*(nmax+1)/2, double)) ||
        !(x = NAG_ALLOC(nsum * p, double)) ||
        !(y = NAG_ALLOC(nsum, double)) ||
        !(zin = NAG_ALLOC(nmax, double)) ||
        !(irank = NAG_ALLOC(nmax, Integer)) )
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
#ifdef NAG_COLUMN_MAJOR
    pdx = nsum;
    pdparvar = p+1;
#else
    pdx = p;
    pdparvar = p;
#endif

    /* Read in observations and design matrices for each sample. */
    for (i = 1; i <= nsum; ++i)
    {
        Vscanf("%lf", &y[i - 1]);
        for (j = 1; j <= p; ++j)
            Vscanf("%lf", &X(i,j));
    }
    Vscanf("%*[\n] ");

    g08rac(order, ns, nv, y, p, x, pdx, idist, nmax, tol,
           parvar, pdparvar, irank, zin, eta, vapvec, parest, &fail);
    if (fail.code != NE_NOERROR)
    {
        Vprintf("Error from g08rac.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    Vprintf("\n");
    Vprintf("Score statistic\n");
    for (i = 1; i <= p; ++i)
        Vprintf("%9.3f%s", parest[i - 1], i%2 == 0 || i == p ? "\n":"" );
    Vprintf("\n");

```

```

Vprintf("Covariance matrix of score statistic\n");
for (j = 1; j <= p; ++j)
{
    for (i = 1; i <= j; ++i)
        Vprintf("%9.3f%s", PARVAR(i,j), i%2 == 0 || i == j ? "\n":" ");
}
Vprintf("\n");

Vprintf("Parameter estimates\n");
for (i = 1; i <= p; ++i)
    Vprintf("%9.3f%s", parest[p + i - 1], i%2 == 0 || i == p ? "\n":" ");
Vprintf("\n");

Vprintf("Covariance matrix of parameter estimates\n");
for (i = 1; i <= p; ++i)
{
    Vprintf(" ");

    for (j = 1; j <= i; ++j)
        Vprintf("%9.3f%s", PARVAR(i + 1,j), j%2 == 0 || j == i ? "\n":" ");
}
Vprintf("\n");

Vprintf("Chi-squared statistic =%9.3f with%2ld d.f.\n",
        parest[p * 2], p);
Vprintf("\n");
Vprintf("Standard errors of estimates and\n");
Vprintf("approximate z-statistics\n");
for (i = 1; i <= p; ++i)
    Vprintf("%9.3f%14.3f\n", parest[2*p + 1 + i - 1], parest[p * 3 + 1 + i -
1]);
Vprintf("\n");
}
END:
if (eta) NAG_FREE(eta);
if (parest) NAG_FREE(parest);
if (parvar) NAG_FREE(parvar);
if (vapvec) NAG_FREE(vapvec);
if (x) NAG_FREE(x);
if (y) NAG_FREE(y);
if (zin) NAG_FREE(zin);
if (irank) NAG_FREE(irank);
if (nv) NAG_FREE(nv);

return exit_status;
}

```

## 9.2 Program Data

g08rac Example Program Data

```

1 2 2 0.00001
20
1.0 1.0 23.0
1.0 1.0 32.0
3.0 1.0 37.0
4.0 1.0 41.0
2.0 1.0 41.0
4.0 1.0 48.0
1.0 1.0 48.0
5.0 1.0 55.0
4.0 1.0 55.0
4.0 0.0 56.0
4.0 1.0 57.0
4.0 1.0 57.0
4.0 1.0 57.0
1.0 0.0 58.0
4.0 1.0 59.0
5.0 0.0 59.0
5.0 0.0 60.0

```

```
4.0 1.0 61.0
4.0 1.0 62.0
3.0 1.0 62.0
```

### 9.3 Program Results

g08rac Example Program Results

```
Number of samples = 1
Number of parameters fitted = 2
Distribution = 2
Tolerance for ties = 0.00001
```

```
Score statistic
-1.048    64.333
```

```
Covariance matrix of score statistic
  0.673
-4.159   533.670
```

```
Parameter estimates
-0.852    0.114
```

```
Covariance matrix of parameter estimates
  1.560
  0.012    0.002
```

```
Chi-squared statistic =    8.221 with 2 d.f.
```

```
Standard errors of estimates and
approximate z-statistics
  1.249    -0.682
  0.044    2.567
```

---