

## nag\_tsa\_auto\_corr (g13abc)

### 1. Purpose

**nag\_tsa\_auto\_corr (g13abc)** computes the sample autocorrelation function of a time series. It also computes the sample mean, the sample variance and a statistic which may be used to test the hypothesis that the true autocorrelation function is zero.

### 2. Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_auto_corr(double x[], Integer nx, Integer nk, double *mean,
                      double *var, double r[], double *stat, NagError *fail)
```

### 3. Description

The data consist of  $n$  observations  $x_i$ , for  $i = 1, 2, \dots, n$  from a time series. The quantities calculated are:

- (a) The sample mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

- (b) The sample variance (for  $n \geq 2$ )

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n - 1)}$$

- (c) The sample autocorrelation coefficients of lags  $k = 1, 2, \dots, K$ , where  $K$  is a user-specified maximum lag, and  $K < n$ ,  $n > 1$ .

The coefficient of lag  $k$  is defined as

$$r_k = \frac{\sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

See page 496 *et seq.* of Box and Jenkins (1976) for further details.

- (d) A test statistic defined as

$$\mathbf{stat} = n \sum_{k=1}^K r_k^2,$$

which can be used to test the hypothesis that the true autocorrelation function is identically zero.

If  $n$  is large and  $K$  is much smaller than  $n$ , **stat** has a  $\chi_K^2$  distribution under the hypothesis of a zero autocorrelation function. Values of **stat** in the upper tail of the distribution provide evidence against the hypothesis.

Section 8.2.2 of Box and Jenkins (1976) provides further details of the use of **stat**.

#### 4. Parameters

**x[nx]**

Input: the time series,  $x_i$ , for  $i = 1, 2, \dots, n$ .

**nx**

Input: the number of values,  $n$ , in the time series.

Constraint:  $\mathbf{nx} > 1$ .

**nk**

Input: the number of lags,  $K$ , for which the autocorrelations are required. The lags range from 1 to  $K$  and do not include zero.

Constraint:  $0 < \mathbf{nk} < \mathbf{nx}$ .

**mean**

Output: the sample mean of the input time series.

**var**

Output: the sample variance of the input time series.

**r[nk]**

Output: the sample autocorrelation coefficient relating to lag  $k$ , for  $k = 1, 2, \dots, K$ .

**stat**

Output: the statistic used to test the hypothesis that the true autocorrelation function of the time series is identically zero.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

#### 5. Error Indications and Warnings

**NE\_INT\_ARG\_LE**

On entry,  $\mathbf{nx}$  must not be less than or equal to 1:  $\mathbf{nx} = \langle \text{value} \rangle$ .

On entry,  $\mathbf{nk}$  must not be less than or equal to 0:  $\mathbf{nk} = \langle \text{value} \rangle$ .

**NE\_2\_INT\_ARG\_LE**

On entry  $\mathbf{nx} = \langle \text{value} \rangle$  while  $\mathbf{nk} = \langle \text{value} \rangle$ . These parameters must satisfy  $\mathbf{nx} > \mathbf{nk}$ .

**NE\_TIME\_SERIES\_IDEN**

On entry, all values of  $\mathbf{x}$  are practically identical, giving zero variance. In this case  $\mathbf{r}$  and  $\mathbf{stat}$  are undefined on exit.

#### 6. Further Comments

The time taken by the routine is approximately proportional to  $\mathbf{nx} \times \mathbf{nk}$ .

##### 6.1. Accuracy

The computations are believed to be stable.

##### 6.2. References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control (Revised Edition)*  
Holden-Day.

#### 7. See Also

nag\_tsa\_auto\_corr\_part (g13acc)

#### 8. Example

In the example below, a set of 50 values of sunspot counts is used as input. The first 10 autocorrelations are computed.

## 8.1. Program Text

```

/* nag_tsa_auto_corr(g13abc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <naggi3.h>

#define NXMAX 50
#define NKMAX 10

main()
{
    double mean, xv, stat;
    Integer i, nk, nx;
    double r[NKMAX], x[NXMAX];

    Vprintf("g13abc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[\n]");
    Vscanf("%ld %ld", &nx, &nk);

    if (nk>0 && nk<=NKMAX && nx>0 && nx<=NXMAX)
    {
        for (i=0; i<nx; ++i)
            Vscanf("%lf", &x[i]);
        Vprintf("\nThe first %2ld coefficients are required\n", nk);

        g13abc(x, nx, nk, &mean, &xv, r, &stat, NAGERR_DEFAULT);

        Vprintf("The input array has sample mean %12.4f\n", mean);
        Vprintf("The input array has sample variance %12.4f\n", xv);
        Vprintf("The sample autocorrelation coefficients are\n\n");
        Vprintf("  Lag      Coeff\n");
        for (i=0; i<10; ++i)
            Vprintf("%6ld%10.4f\n", i+1, r[i]);
        Vprintf("\nThe value of stat is %12.4f\n", stat);
    }
    else
    {
        Vfprintf(stderr, "One or both of nk and nx are out of range:\n
nk = %-3ld while nx = %-3ld\n", nk, nx);
        exit(EXIT_FAILURE);
    }
    exit(EXIT_SUCCESS);
}

```

## 8.2. Program Data

```

g13abc Example Program Data
50 10
 5.0 11.0 16.0 23.0 36.0
58.0 29.0 20.0 10.0 8.0
 3.0 0.0 0.0 2.0 11.0
27.0 47.0 63.0 60.0 39.0
28.0 26.0 22.0 11.0 21.0
40.0 78.0 122.0 103.0 73.0
47.0 35.0 11.0 5.0 16.0
34.0 70.0 81.0 111.0 101.0
73.0 40.0 20.0 16.0 5.0
11.0 22.0 40.0 60.0 80.9

```

### 8.3. Program Results

g13abc Example Program Results

The first 10 coefficients are required  
The input array has sample mean 37.4180  
The input array has sample variance 1002.0301  
The sample autocorrelation coefficients are

Lag	Coeff
1	0.8004
2	0.4355
3	0.0328
4	-0.2835
5	-0.4505
6	-0.4242
7	-0.2419
8	0.0550
9	0.3783
10	0.5857

The value of stat is 92.1231

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